Problem 8

Sketch the region in the plane consisting of all points (x, y) such that

$$|x - y| + |x| - |y| \leq 2$$

Solution

The aim in this problem is to consider portions of the xy-plane, one by one, for which this inequality simplifies to one that can be solved. Specifically, the xy-plane will be partitioned into the four quadrants as well as above and below the line $y = x$.

Quadrant 1

Within this quadrant, $x \geq 0$ and $y \geq 0$. Below the line, $y \leq x$, or $x - y \geq 0$.

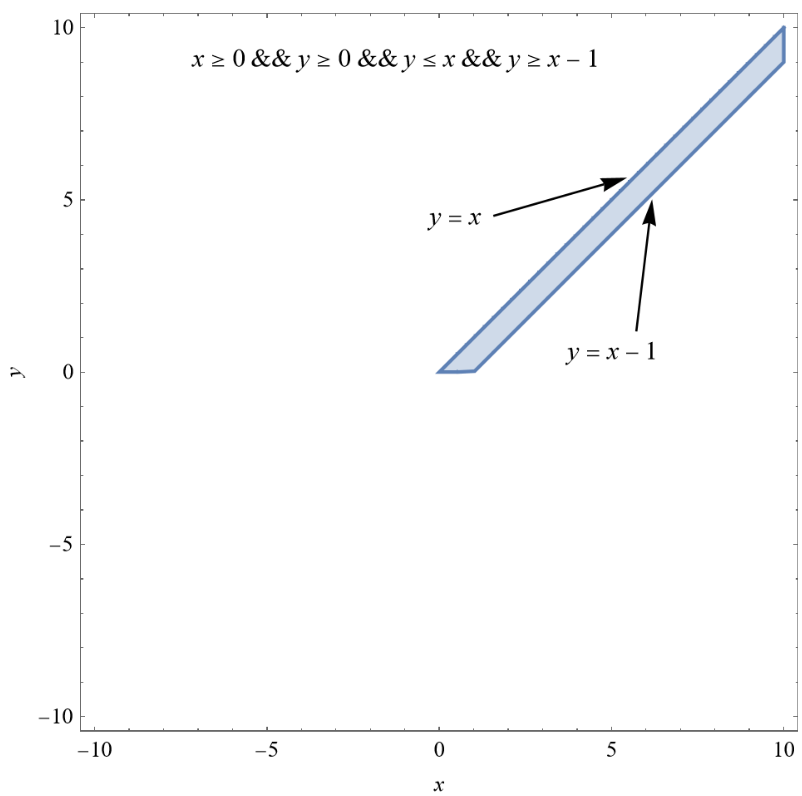
$$|x - y| + |x| - |y| \leq 2$$

$$(x - y) + (x) - (y) \leq 2$$

$$2x - 2y \leq 2$$

$$x - y \leq 1$$

$$y \geq x - 1$$



Above the line, $y \geq x$, or $x - y \leq 0$.

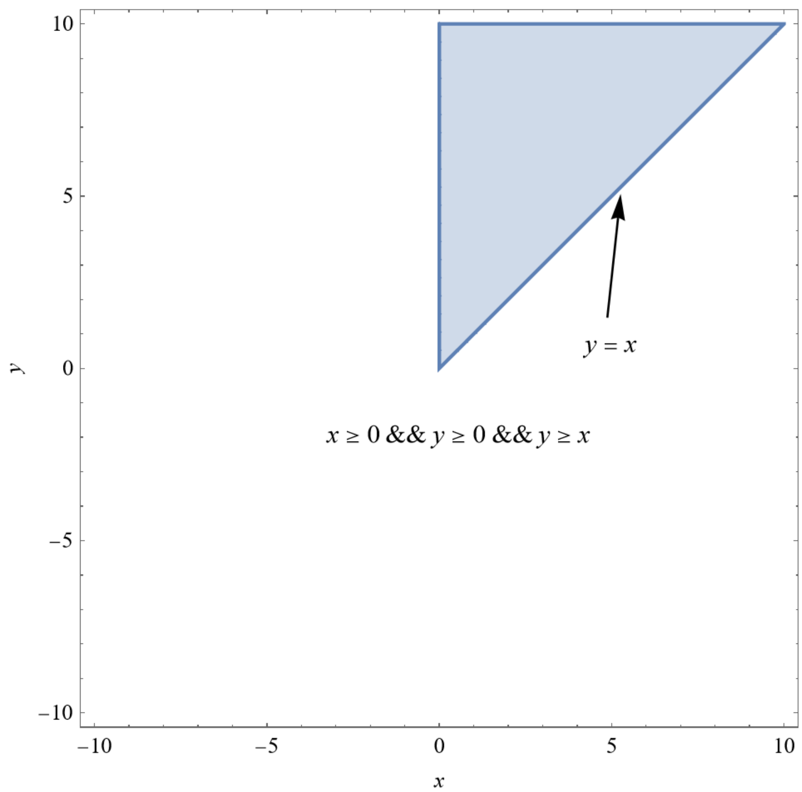
$$|x - y| + |x| - |y| \leq 2$$

$$-(x - y) + (x) - (y) \leq 2$$

$$-x + y + x - y \leq 2$$

$$0 \leq 2$$

This is a true statement regardless of what x and y are, so all points on and above the line $y = x$ satisfy the conditions.



Quadrant 2

Within this quadrant, $x \leq 0$ and $y \geq 0$ and $y \geq x$, or $x - y \leq 0$.

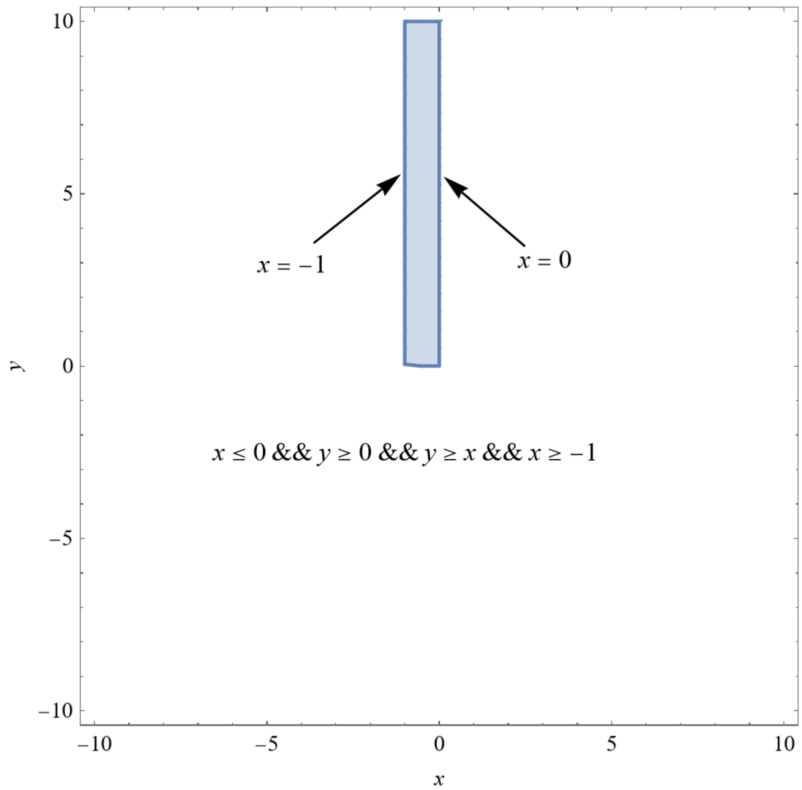
$$|x - y| + |x| - |y| \leq 2$$

$$-(x - y) + (-x) - (y) \leq 2$$

$$-x + y - x - y \leq 2$$

$$-2x \leq 2$$

$$x \geq -1$$



Quadrant 3

Within this quadrant, $x \leq 0$ and $y \leq 0$. Below the line, $y \leq x$, or $x - y \geq 0$.

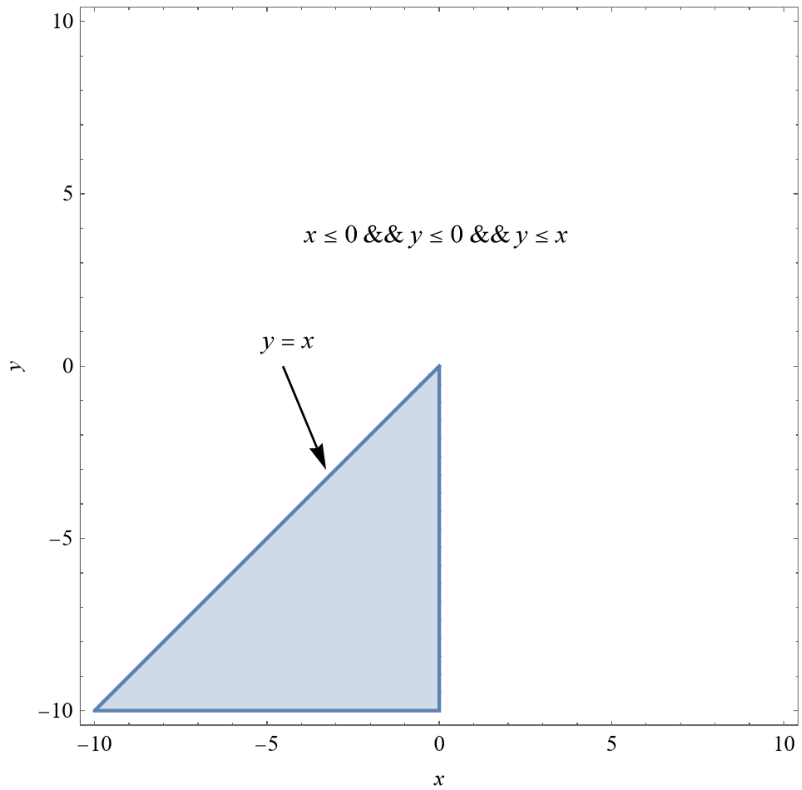
$$|x-y| + |x| - |y| \leq 2$$

$$(x-y) + (-x) - (-y) \leq 2$$

$$x - y - x + y \leq 2$$

$$0 \leq 2$$

This is a true statement regardless of what x and y are, so all points on and below the line $y = x$ satisfy the conditions.



Above the line, $y \geq x$, or $x - y \leq 0$.

$$|x - y| + |x| - |y| \leq 2$$

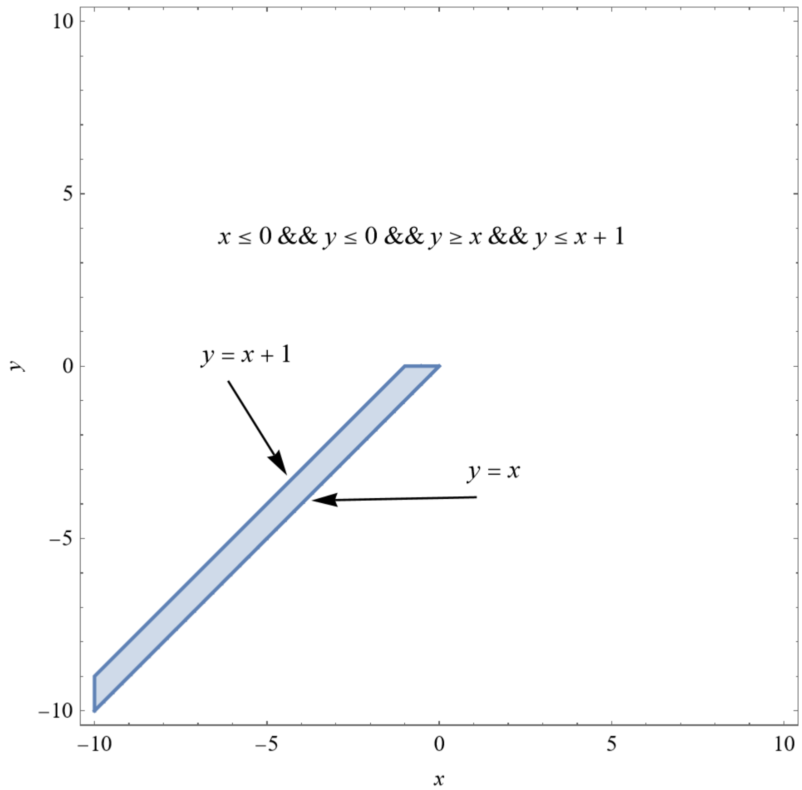
$$-(x - y) + (-x) - (-y) \leq 2$$

$$-x + y - x + y \leq 2$$

$$2y - 2x \leq 2$$

$$y - x \leq 1$$

$$y \leq x + 1$$



$x \le 0$ && $y \le 0$ && $y \ge x$ && $y \le x + 1$
$y = x + 1$
$y = x$
10
5
0
−5
−10
−10
−5
0
5
10
x
y

Quadrant 4

Within this quadrant, $x \geq 0$ and $y \leq 0$ and $y \leq x$, or $x - y \geq 0$.

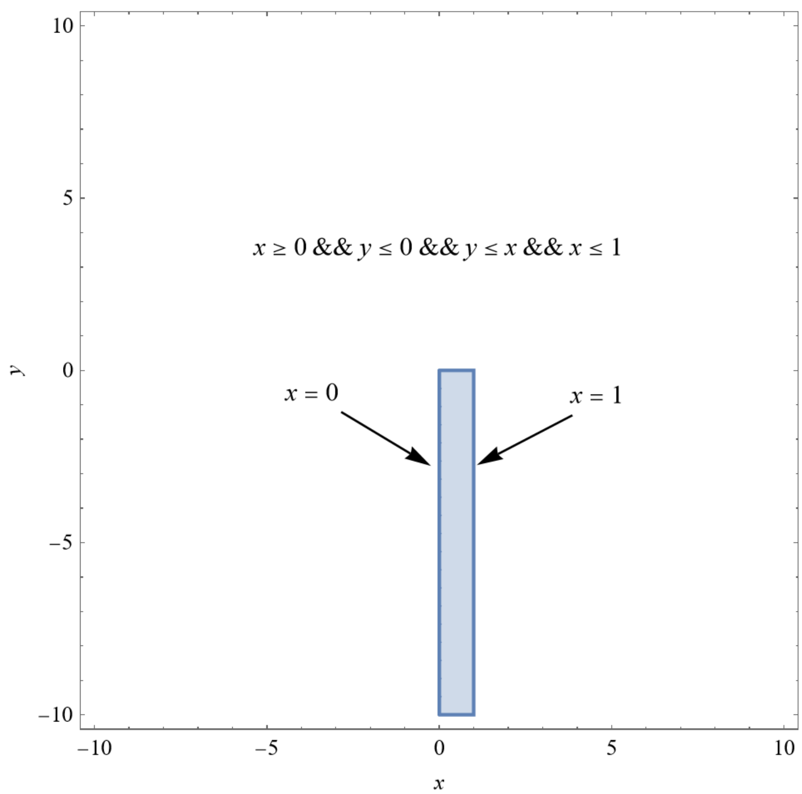
$$|x - y| + |x| - |y| \leq 2$$

$$(x - y) + (x) - (-y) \leq 2$$

$$x - y + x + y \leq 2$$

$$2x \leq 2$$

$$x \leq 1$$



Superimpose all of these graphs to get all of the points in the xy-plane that satisfy the original inequality.